to lattice matching. Since the [0 1 0] triclinic dimension (parallel to the substrate) is 21.4 Å, with five chains equally spaced along that dimension, a spacing of 4.3 Å would be most favourable according to lattice matching rules. Of all the alkali halide substrates, NaCl and KCl, with [1 1 0] spacings of 4.0 and 4.5 Å, respectively, should yield the maximum growth. This is precisely what is observed.

It can be concluded that isotactic polypropylene crystallized epitaxially in the unusual triclinic crystal form on alkali halides at 160° C. Epitaxial growth at 140° C produced monoclinic epitaxial crystals. The alkali halide substrate permitted crystallization at a high enough temperature to form the γ structure using very slow crystallization rates. Epitaxially grown crystals on NaCl and KCl at 160° C exhibited branch growth with an unusual morphology, and the nucleation density was definitely influenced by a matching of polymer to substrate dimensions.

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Fracture of brittle materials in uniaxial compression

It is well known that brittle materials are often used as structural materials in compression, rather than in tension, since tension gives rise to catastrophic failures. In this paper a theory is proposed for the mechanism of failure of a brittle material in uniaxial compression.

The strength and direction of the propagation of an inclined crack (see Fig. 1) in a brittle material, under a uniaxial loading system, have been carried out [1, 2] using strain energy density concepts. The axial stress, σ, required to propagate an inclined crack (see Fig. 1) is given by

$$\sigma^2 a = \frac{2}{\pi} K_{\mathrm{IC}}{}^2 \mathrm{U}(\beta), \qquad (1)$$

where K_{IC} is the critical stress intensity factor, a is the semi-crack length and $\mathrm{U}(\beta)$ is a function of crack angle, β, which has two values corresponding to tensile and compressive loadings.

Under a tensile loading system, propagation of a single crack leads to total failure, since the subsequent crack path runs normal to the applied stress. Therefore the strength in tension has been studied [3, 4] using the "weakest link concept". This study assumed that the probability density, $f(a)$, of the semi-crack length is well fitted by

$$f(a) = \frac{c^{n-1} a^{-n} e^{-c/a}}{(n-2)!} \text{ for } a > 0, \qquad (2)$$

where c/n is the mode of the distribution and n determines the rate at which the density tends to zero with increase in crack size. It is then shown that the distribution of strengths (tensile) is closely fitted by a Weibull distribution [5] for sufficiently large volumes and that the Weibull modulus, m, is related to n which is a property of the flaw size distribution, by the expression

$$m = 2n - 2 \qquad (3)$$

In the case of uniaxial compression the subsequent crack path does not grow catastrophically,

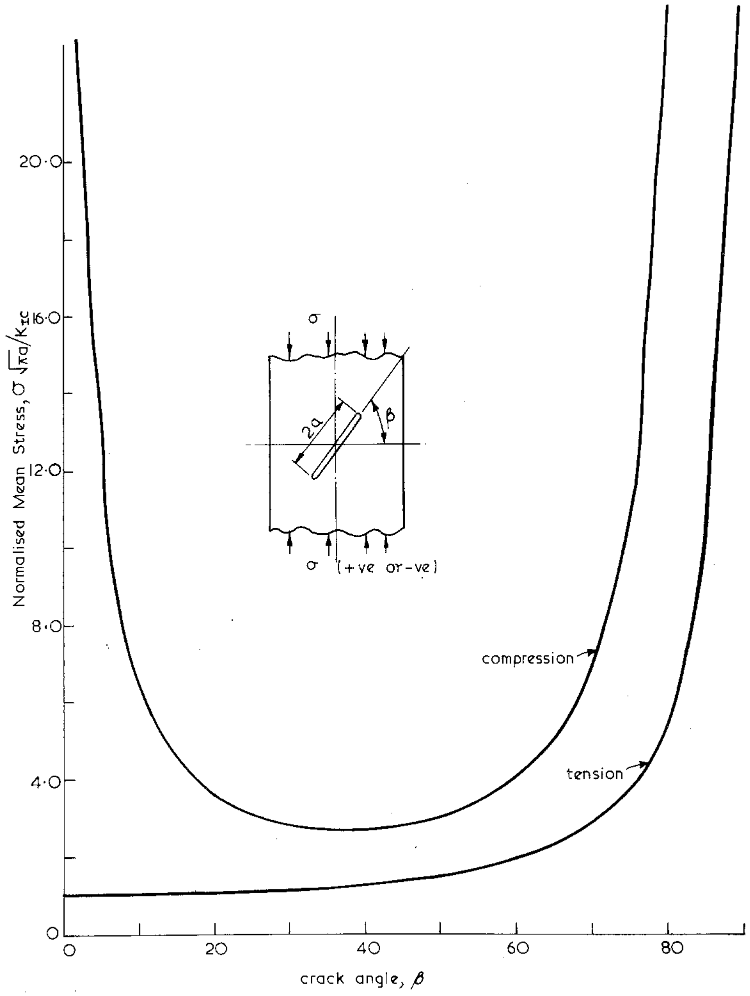

Figure 1 Strength of an inclined crack under uniaxial compression and tension.

but grows only up to a certain length on gradual loading [6] since it tends to align itself along the direction of the applied load thereby requiring a very high stress to propagate it further (see Fig. 1). It follows that in compression the failure of a single crack does not lead to total failure of the material. The model proposed is that the final failure of a brittle material, under compression, only occurs after a certain proportion of the cracks have failed, where the proportion will be a material property. Although the proportion will in practice be variable, the model assumes that the variance of the proportion is small for a large number of cracks. The splitting of the brittle material then occurs when several of the failed cracks join together to form the fracture surface and this is often observed in uniaxial compression tests.

Assuming that all orientations of cracks are equally likely, the probability of failure, $F(\sigma)$, of one crack at stress, σ, is given by

$$F(\sigma) = \int_0^{\pi/2} \int_z^{\infty} \frac{2}{\pi} f(a)\,\mathrm{d}a\,\mathrm{d}\beta \qquad (4)$$

where $z = 2K_{\mathrm{IC}}^2 \mathrm{U}(\beta)/\pi\sigma^2$. On substituting for $f(a)$ from Equation 2 and using the computed values of $\mathrm{U}(\beta)$, $F(\sigma)$ can be evaluated by a combination of analytical and numerical methods. From the theory of order statistics it follows that, for the proposed model, the probability of material failure, P_{f}, at stress σ is given by

$$P_f = \int_0^\sigma \frac{N!}{(N_f-1)!(N-N_f)!}[F(\sigma)]^{N_f-1} \times [1-F(\sigma)]^{N-N_f}\frac{dF}{d\sigma}\,d\sigma \qquad (5)$$

where a proportion of N_f/N out of a total of N cracks have failed. The expression for P_f can be evaluated numerically for different combinations of N and N_f/N, and the average strength, $\bar{\sigma}$, is then computed from

$$\bar{\sigma} = \int_0^\infty (1-P_f)d\sigma. \qquad (6)$$

For large N, the sampling distribution of σ is approximately normal with mean μ given by $F(\mu) = N_f/(N+1)$ and variance inversely proportional to N. This is confirmed by the numerical computations of Equations 5 and 6. Table I shows that the average compressive strength is fairly independent of the number of cracks for a given proportion. Thus if the proposed model for the failure of a brittle material under compression is realistic, then the observed compressive strength of a given material should closely follow a normal distribution with mean independent of volume and variance inversely proportional to volume, as the volume is proportional to the number of cracks in a given material. Experimental observations on the compressive strength of two brittle materials support the theoretical predictions.

The above model enables the compressive strength to be related to the tensile strength provided that the Weibull modulus, the number of cracks per unit volume and the exact proportion of cracks that should fail prior to material failure are known. This information may be experimentally forthcoming, in the near future, using acoustic emissions devices. The theory described can also be applied to biaxial systems by using the appropriate value of $U(\beta)$. For an engineer, who is interested in designing with a brittle material, it should in the future be possible to derive "safety factors" on the basis of the probabilities of failure.

TABLE I Normalized average compressive stress, $\bar{\sigma}_c/\sigma_I$, for $N_f/N = 2\%$ and 8%. $\bar{\sigma}_T/\sigma_I$ is the normalized average tensile strength based on the failure of one crack. $\sigma_I = K_{IC}/\sqrt{\pi c}$.

N	$m = 2 (n = 2)$			$m = 10 (n = 6)$		
	$\bar{\sigma}_T/\sigma_I$	$\bar{\sigma}_c/\sigma_I$		$\bar{\sigma}_T/\sigma_I$	$\bar{\sigma}_c/\sigma_I$	
		2%	8%		2%	8%
200	0.0870	0.558	1.174	1.145	3.996	5.060
400	0.0615	0.567	1.179	1.051	4.025	5.069
600	0.0502	0.570	1.181	1.001	4.034	5.073
1000	0.0389	0.573	1.182	0.941	4.042	5.075

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Fritted thick film conductor adherence: role of firing atmosphere

Previous studies showed that the adherence of a fritted Pt/Au thick film conductor fired in air on alumina substrates is a function of the thick film peak firing temperature and time and the character of the alumina substrate [1, 2]. In these studies, a fracture energy approach was used to determine thick film adherence as well as to relate this to the thick film-substrate microstructure. It was demonstrated that the above parameters altered the formation of the interpenetrating thick film glass–metal interface required for maximum adherence by affecting the thick film metal sintering and glass flow behaviour.

Besides the above parameters, changes in the thick film firing atmosphere may also be a